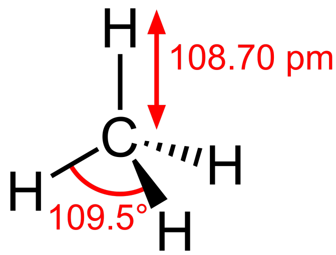
Exam Symmetry in Physics

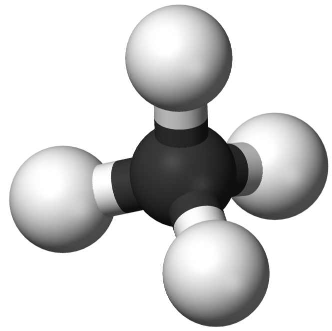
Date	June 22, 2021
Time	9:00 - 11:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- All subquestions (a, b, etc) of the three exercises have equal weight
- Illegible answers will be not be graded
- Good luck!

Exercise 1

Consider the methane molecule CH_4 depicted in the figures.





(a) Identify all rotations that leave the methane molecule invariant. These symmetry transformations form the group G_{M}. Note that reflections do not need to be considered.

(b) Divide the elements of G_{M} into conjugacy classes using geometric arguments and construct the character table of G_{M}. Explain how the entries are obtained.

(c) Show whether G_{M} allows in principle for a permanent electric dipole moment, a permanent magnetic dipole moment or neither. Explain how the answer can be understood from the molecule and its symmetry transformations.

Exercise 2

Consider the symmetric group S_3 consisting of the permutations of three objects and view the three basis vectors of R^3 as the three objects that are permuted. This leads to the following three-dimensional rep D^L of S_3:

$$D^L(c) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad D^L(b) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Here the rep is only specified for the two generators c and b of S_3 which in cycle notation are given by $c = (123)$ and $b = (12)$.

(a) Show that D^L indeed forms a representation of S_3 (for example by using the presentation of the group).

(b) Show using Schur's lemma that D^L is not an irrep of S_3 and decompose D^L into irreps of S_3 using the character table of S_3.

(c) Decompose $D^L \otimes D^L$ into irreps of S_3 using the character table of S_3.

Exercise 3

Consider the group $O(2)$ of real orthogonal 2×2 matrices and the group $U(1)$ of unitary 1×1 matrices.

(a) Write down explicitly all elements of $O(2)$ in its defining representation.

(b) Construct a homomorphism from $O(2)$ to $U(1)$ and determine its kernel.

(c) Consider two vectors in R^2: $\vec{v} = (v_x, v_y)$ and $\vec{w} = (w_x, w_y)$. Demonstrate that the quantity $v_x w_y - v_y w_x$ behaves like a pseudoscalar in R^2.